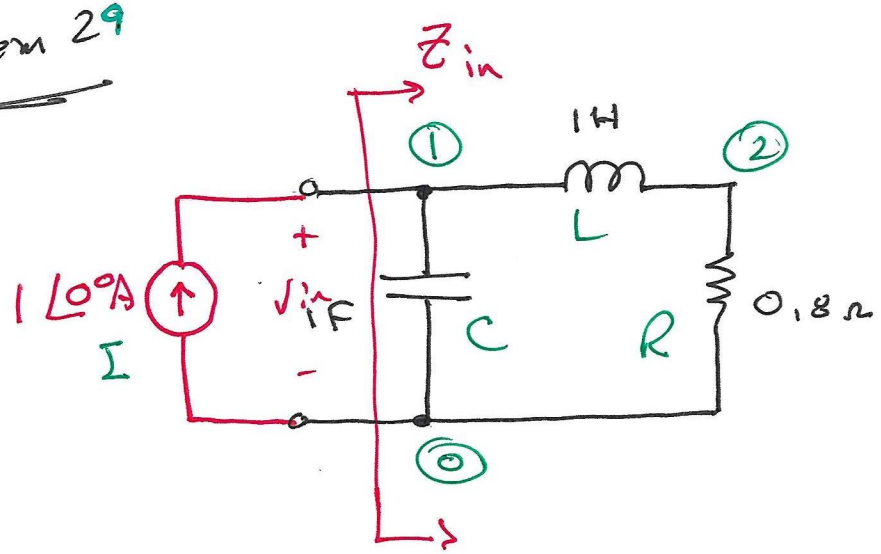
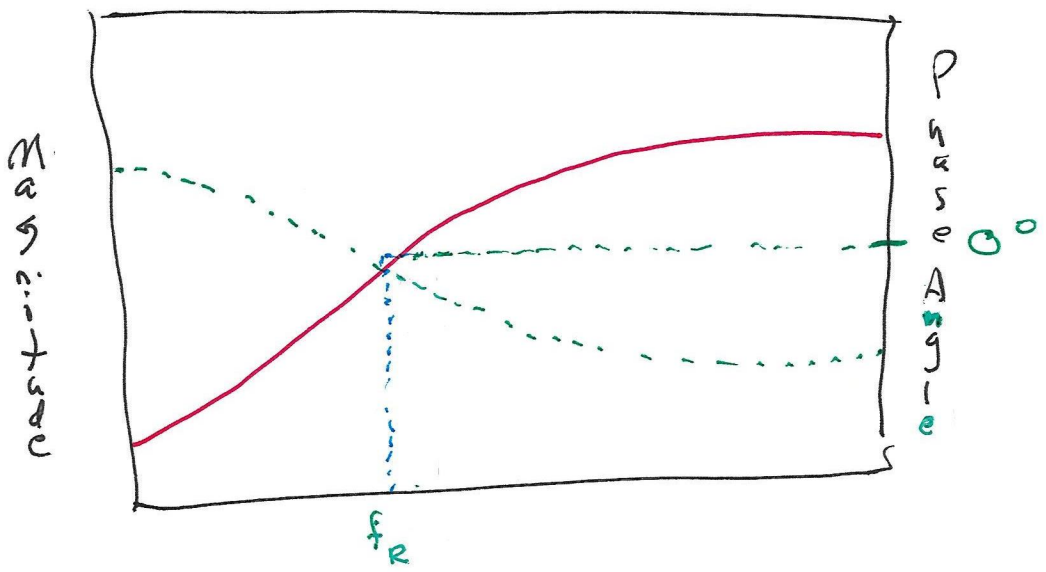
Problem 29



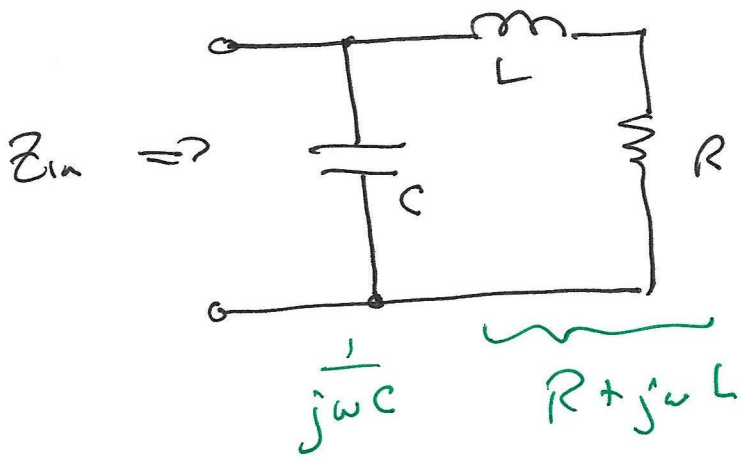
$$V_{in} = Z_{in}(1\angle 0^\circ) = Z_{in}$$

```
Title
L   1   2   1
C   1   0   1
R   2   0   0.8
I   0   1   AC   1   0
.AC  LIN   1000   1   100
.end
```

2nd choice: `.AC DEC 100 1 1G`



$$\omega_R = 2\pi f_R$$



$$Z_{in} = \frac{\frac{1}{j\omega C}(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L} = \frac{R + j\omega L}{1 + j\omega RC - \omega^2 LC}$$

$$= \frac{R + j\omega L}{(1 - \omega^2 LC) + j\omega RC} \cdot \frac{(1 - \omega^2 LC) - j\omega RC}{(1 - \omega^2 LC) - j\omega RC}$$

$$= \frac{R(1 - \omega^2 LC) + j\omega L(1 - \omega^2 LC) - j\omega R^2 C + \omega^2 RLC}{(1 - \omega^2 LC)^2 + (\omega RC)^2}$$

At resonance, the imaginary part of the numerator is zero.

$$\omega L(1-\omega^2 LC) - \omega R^2 C = 0$$

$$L - \omega^2 L^2 C - R^2 C = 0$$

$$\omega^2 L^2 C = L - R^2 C$$

$$\omega^2 = \frac{L - R^2 C}{L^2 C}$$

$$\omega_R = \sqrt{\frac{L - R^2 C}{L^2 C}}$$

$$= \sqrt{\frac{1 - 0.64}{1}}$$

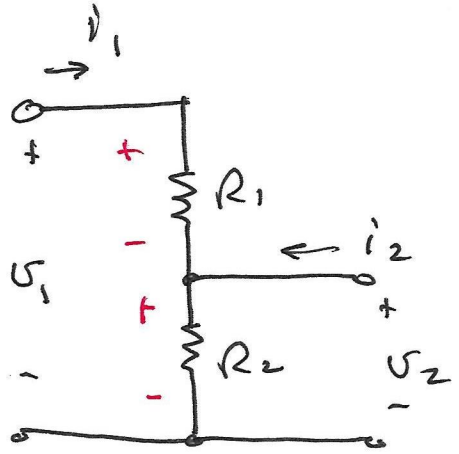
$$= \sqrt{.36}$$

$$= .6$$

Impedance Parameters for a Two-Port Network

$$v_1 = z_{11} i_1 + z_{12} i_2$$

$$v_2 = z_{21} i_1 + z_{22} i_2$$



$$v_1 = z_{11} i_1 + z_{12} i_2$$

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2 = 0} = R_1 + R_2$$

$$v_2 = z_{21} i_1 + z_{22} i_2$$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2 = 0} = R_2$$

$$v_1 = z_{11} i_1 + z_{12} i_2$$

$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = R_2$$

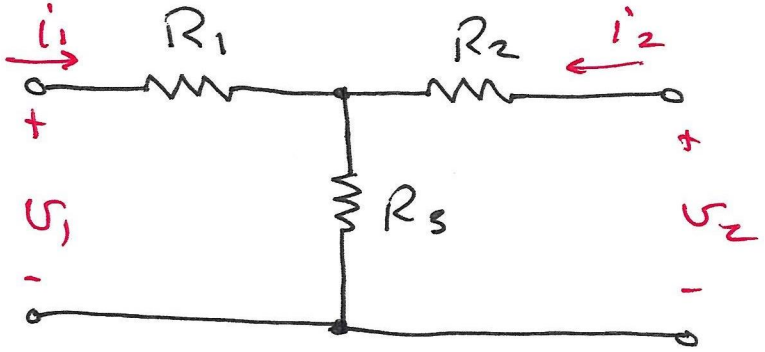
$$v_2 = z_{21} i_1 + z_{22} i_2$$

$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = R_2$$

$\therefore$

$$v_1 = \overbrace{(R_1 + R_2)}^{z_{11}} i_1 + \overbrace{R_2}^{z_{12}} i_2$$

$$v_2 = \underbrace{R_2}_{z_{21}} i_1 + \underbrace{R_2}_{z_{22}} i_2$$



Find the impedance parameters $z_{11}, z_{12}, z_{21}, z_{22}$

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = \left.\frac{(R_1+R_3)\, i_1}{i_1}\right|_{i_2=0} = R_1 + R_3$$

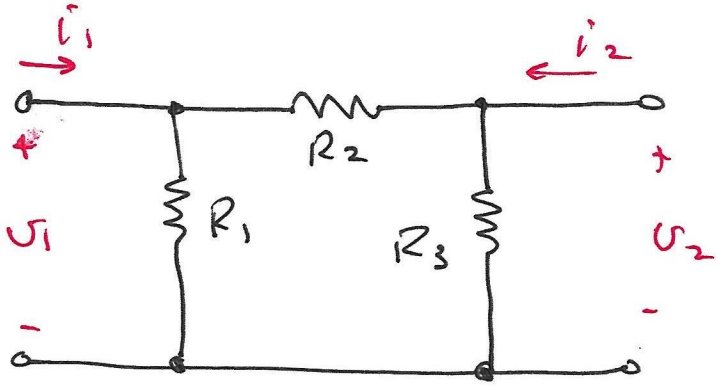
$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = \left.\frac{R_3 i_2}{i_2}\right|_{i_1=0} = R_3$$

$\therefore$ $\boxed{v_1 = (R_1 + R_3) i_1 + R_3 i_2}$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = \left.\frac{R_3\, i_1}{i_1}\right|_{i_2=0} = R_3$$

$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = \left.\frac{(R_2+R_3)\, i_2}{i_2}\right|_{i_1=0} = R_2+R_3$$

$$\boxed{v_2 = R_3\, i_1 + (R_2+R_3)\, i_2}$$



Determine the impedance parameters.

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = \left.\frac{[R_1 \| (R_2+R_3)]\, i_1}{i_1}\right|_{i_2=} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$$

$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = \frac{R_3 R_1}{R_1+R_2+R_3}\left.\frac{i_2}{i_2}\right|_{i_1=0} = \frac{R_1 R_3}{R_1+R_2+R_3}$$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = \frac{R_1 R_3}{R_1+R_2+R_3}\left.i_1\right|_{i_2=0} = \frac{R_1 R_3}{R_1+R_2+R_3}$$

$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = \frac{R_1+R_2}{R_1+R_2+R_3}\frac{i_2}{i_2}R_3 = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$$